TreeFlow: Going Beyond Tree-based Parametric Probabilistic Regression

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Motivation

- Fact #1: Tree-based ensembles excel in classification and regression with mixed-type variable tabular data, e.g., CatBoost.
- Fact #2: Current approaches use Gaussian or parametric distributions for uncertainty modeling, e.g., CatBoost, NGBoost, PGBM.
- Fact #3: Existing methods struggle to handle multi-modal distributions and do not support high-dimensional probabilistic predictions.

Here comes the TreeFlow!

Method



- Regression model for tabular data
- Numerical and categorical data
- Univariate and multivariate targets
- Non-Gaussian, non-parametric distributions
- **Probabilistic** and **deterministic** predictions



Tree-based Feature Extractor - extract the vector of binary features from the structure of the tree-based ensemble model. Shallow Feature Extractor - a shallow neural network; maps highdimensional binary vectors to low-dimensional feature space. **Conditional Continuous Normalizing Flow** - takes previous vector as a conditioning factor; models complex probability distribution.

Univariate Flexible Probabilistic Regression



Multivariate Flexible Probabilistic Regression



Univariate Regression on Mixed-type Data

Probabilistic Regression

Univariate Regression on Mixed-type Data

Deterministic Regression

Ð	NLL			\mathbf{CRPS}			\mathbf{RMSE}					
DATASET	CATBOOST	PGBM	TREEFLOW	Catronst	\mathbf{PGBM}	TREEFLOW	DATASET	CatBoost	\mathbf{PGBM}	TREEFLOW(AVG)	TREEFLOW(@1)	TREEFLOW(@2)

Avocado	-0.40 ± 0.01	-0.45 ± 0.01	-0.47 \pm 0.03	0.0992 ± 0.0018	0.0870 ± 0.0013	0.0854 ± 0.0024
BIGMART	-0.05 ± 0.02	$\textbf{-0.10} \pm \textbf{0.02}$	-0.08 ± 0.02	0.1270 ± 0.0021	$\textbf{0.1259} \pm \textbf{0.0023}$	0.1294 ± 0.0027
DIAMONDS	-1.80 ± 0.02	-1.41 ± 0.76	-1.94 \pm 0.03	0.0222 ± 0.0002	0.0447 ± 0.0474	$\boldsymbol{0.0210\pm0.0005}$
Diamonds 2	-1.89 ± 0.02	-1.24 ± 0.83	$\textbf{-2.14}\pm\textbf{0.05}$	0.0217 ± 0.0002	0.0461 ± 0.0504	$\textbf{0.0197} \pm \textbf{0.0005}$
Laptop	-0.89 ± 0.08	-0.97 \pm 0.09	-0.74 ± 0.13	0.0572 ± 0.0049	$\textbf{0.0474} \pm \textbf{0.0034}$	0.0563 ± 0.0043
Pak Wheel	-1.40 ± 0.05	-0.53 ± 0.02	-1.60 \pm 0.03	0.0362 ± 0.0006	0.0813 ± 0.0009	$\textbf{0.0327} \pm \textbf{0.0007}$
Sydney	-0.54 ± 0.04	0.20 ± 1.02	-0.66 \pm 0.01	0.0726 ± 0.0011	0.2383 ± 0.2646	$\textbf{0.0721} \pm \textbf{0.0008}$

Univariate Regression on Numeric Data

Probabilistic Regression

DATASET	DEEP. ENS.	CatBoost	NGBOOST	RoNGBA	PGBM	TREEFLOW
Concrete Energy	3.06 ± 0.18 1.38 ± 0.22	3.06 ± 0.13 1.24 ± 1.28	3.04 ± 0.17 0.60 ± 0.45	2.94 ± 0.18 0.37 ± 0.28	2.75 ± 0.21 1.74 ± 0.04	3.02 ± 0.15 0.85 ± 0.35
KIN8NM	-1.20 ± 0.02	-0.63 ± 0.02	-0.49 ± 0.02	-0.60 ± 0.03	-0.54 ± 0.04	-1.03 ± 0.06
NAVAL	$ $ -5.63 \pm 0.05	-5.39 ± 0.04	-5.34 ± 0.04	-5.49 ± 0.04	-3.44 ± 0.04	-5.54 ± 0.16
Power	2.79 ± 0.04	2.72 ± 0.12	2.79 ± 0.11	2.65 ± 0.08	2.60 ± 0.02	2.65 ± 0.06
Protein	2.83 ± 0.02	2.73 ± 0.07	2.81 ± 0.03	2.76 ± 0.03	2.79 ± 0.01	2.02 ± 0.02
WINE	0.94 ± 0.12	0.93 ± 0.08	0.91 ± 0.06	0.91 ± 0.08	0.97 ± 0.20	$\textbf{-0.56}\pm\textbf{0.62}$
Yacht	1.18 ± 0.21	0.41 ± 0.39	0.20 ± 0.26	1.03 ± 0.44	$\boldsymbol{0.05}\pm\boldsymbol{0.28}$	0.72 ± 0.40
Year MSD	$3.35 \pm \mathrm{NA}$	$3.43 \pm NA$	$3.43 \pm NA$	$3.46 \pm NA$	$3.61 \pm NA$	$3.27\pm\mathbf{NA}$

Avocado	0.1939 ± 0.0043	0.1624 ± 0.0024	0.1676 ± 0.0058	0.1769 ± 0.0087	0.1713 ± 0.0066
BIGMART	0.2284 ± 0.0039	$\boldsymbol{0.2274\pm0.0040}$	0.2335 ± 0.0045	0.2514 ± 0.0087	0.2480 ± 0.0083
DIAMONDS	0.0419 ± 0.0007	0.0403 ± 0.0006	0.0407 ± 0.0009	0.0445 ± 0.0015	$\textbf{0.0343} \pm \textbf{0.0017}$
Diamonds 2	0.0421 ± 0.0006	0.0492 ± 0.0010	0.0398 ± 0.0006	0.0460 ± 0.0014	$\textbf{0.0364} \pm \textbf{0.0004}$
Laptop	0.1028 ± 0.0092	$\textbf{0.0848} \pm \textbf{0.0063}$	0.1014 ± 0.0082	0.1015 ± 0.0076	0.0958 ± 0.0058
Pak Wheel	0.0783 ± 0.0009	0.1630 ± 0.0018	0.0729 ± 0.0018	0.0796 ± 0.0021	$\textbf{0.0654} \pm \textbf{0.0047}$
Sydney	0.1528 ± 0.0057	0.1561 ± 0.0047	0.1518 ± 0.0051	0.1721 ± 0.0041	$\textbf{0.1361} \pm \textbf{0.0066}$

Multivariate Regression on Numeric Data

Probabilistic Regression

DATASET	IND NGBOOST	NGBOOST	TREEFLOW
PARKINSONS	6.86	5.85	5.26
scm20d	94.40	94.81	93.41
WINDTURBINE	-0.65	-0.67	-2.57
Energy	166.90	175.80	180.00
USFLIGHT	9.56	8.57	7.49
Oceanographic	7.74±0.02	$7.73{\pm}0.02$	7.84 ± 0.01